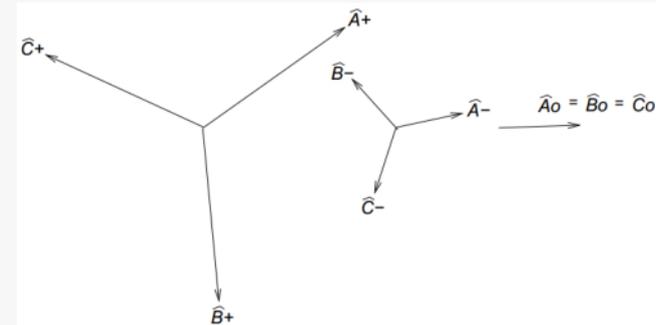


# Sistemas Elétricos de Potência

## Aula 03-P2 – Método das Componentes Simétricas Aplicado aos Equipamentos: Gerador, Transformador e Linhas de Transmissão

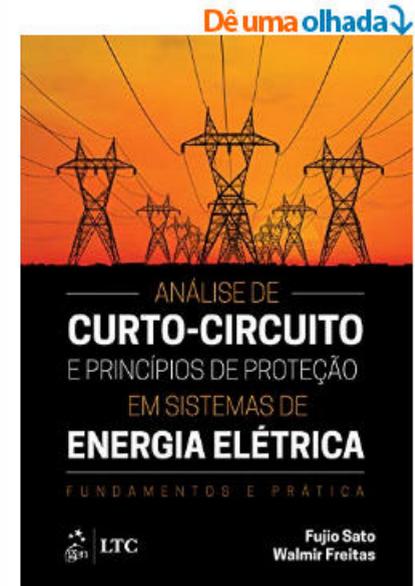


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# Tópicos abordados

- Capítulo 4
- Propriedades da TCS
  - TCS nas fórmulas de potência
  - TCS nos elementos passivos
    - ✓ Linha de Transmissão
    - ✓ Transformadores
    - ✓ Máquina síncrona



## TCS nas fórmulas de potência

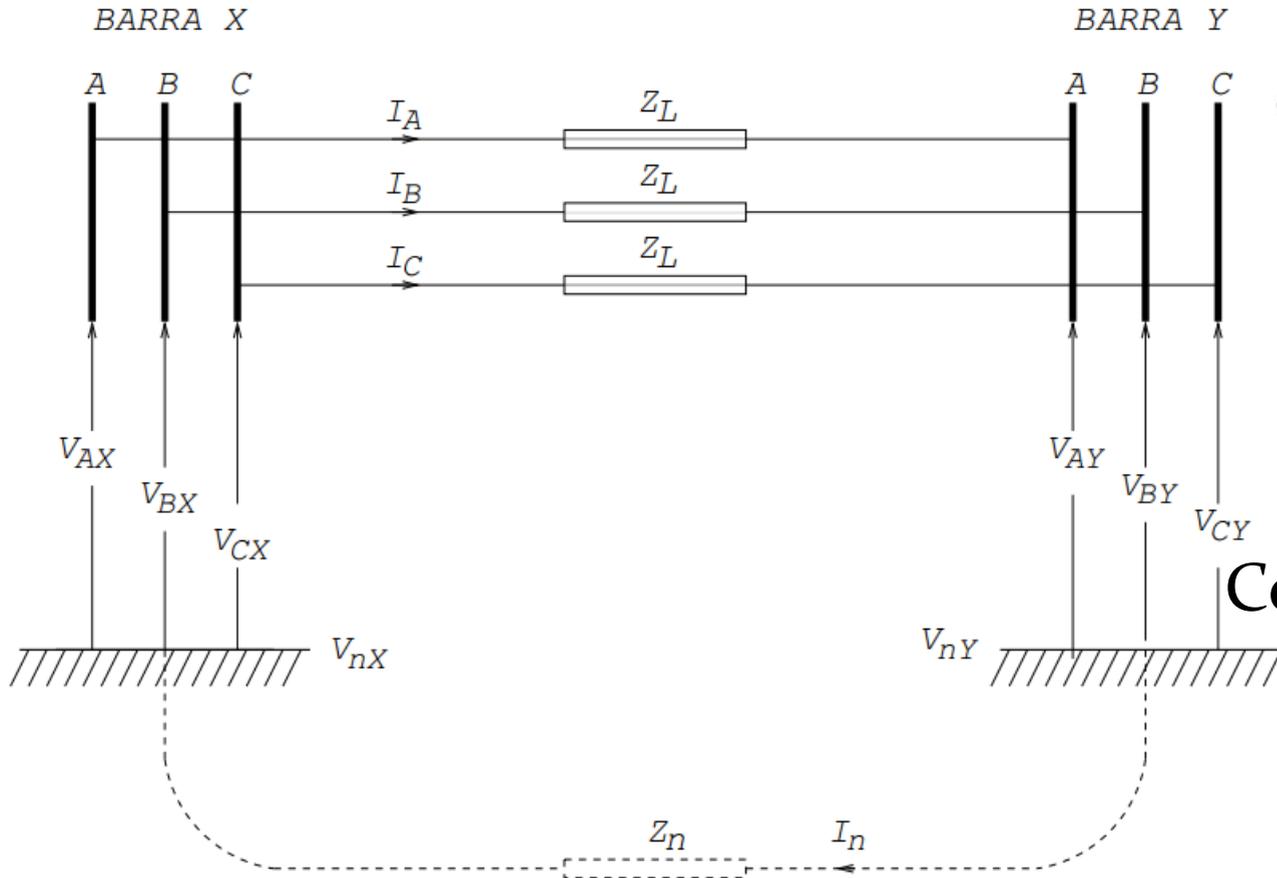
$$S = \hat{V}_A \hat{I}_A^* + \hat{V}_B \hat{I}_B^* + \hat{V}_C \hat{I}_C^* = \underline{V}_P^T \underline{I}_P^*$$

$$S = (T \underline{V}_S)^T (T \underline{I}_S)^* = \underline{V}_S^T (T^T T^*) \underline{I}_S^*$$

$$S = 3 \underline{V}_S^T \underline{I}_S^* = 3 \hat{V}_+ \hat{I}_+^* + 3 \hat{V}_- \hat{I}_-^* + 3 \hat{V}_o \hat{I}_o^*$$

$$S = P + jQ = 3 \hat{V}_+ \hat{I}_+^* + 3 \hat{V}_- \hat{I}_-^* + 3 \hat{V}_o \hat{I}_o^*$$

# Linha de Transmissão



Condições equilibradas

$$V_{nX} = V_{nY} = 0$$

$$I_n = 0$$

Condições desequilibradas

$$I_n = I_A + I_B + I_C$$

## Linha de Transmissão

Condições desequilibradas  $I_n = I_A + I_B + I_C$

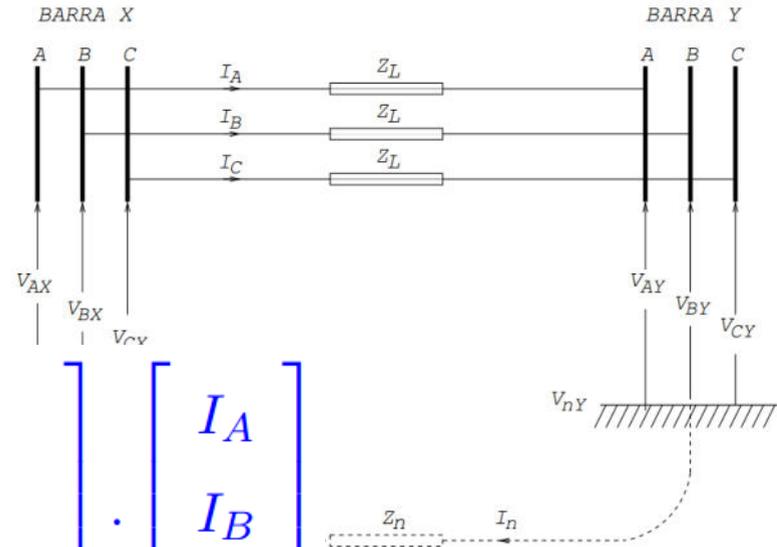
$$V_{AX} - V_{AY} = I_A Z_L + (I_A + I_B + I_C) Z_n$$

$$V_{BX} - V_{BY} = I_B Z_L + (I_A + I_B + I_C) Z_n$$

$$V_{CX} - V_{CY} = I_C Z_L + (I_A + I_B + I_C) Z_n$$

$$\begin{bmatrix} V_{AX} - V_{AY} \\ V_{BX} - V_{BY} \\ V_{CX} - V_{CY} \end{bmatrix} = \begin{bmatrix} Z_L + Z_n & Z_L & Z_L \\ Z_L & Z_L + Z_n & Z_L \\ Z_L & Z_L & Z_L + Z_n \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$\Delta \underline{V}_P = Z_C \underline{I}_P$$



# Linha de Transmissão

## Condições desequilibradas

$$\Delta \underline{V}_P = Z_C \underline{I}_P$$

$$T \Delta \underline{V}_S = Z_C T \Delta \underline{I}_S$$

$$\Delta \underline{V}_S = T^{-1} Z_C T \Delta \underline{I}_S$$

$$Z_S = T^{-1} \begin{bmatrix} Z_L + Z_n & Z_L & Z_L \\ Z_L & Z_L + Z_n & Z_L \\ Z_L & Z_L & Z_L + Z_n \end{bmatrix} T$$

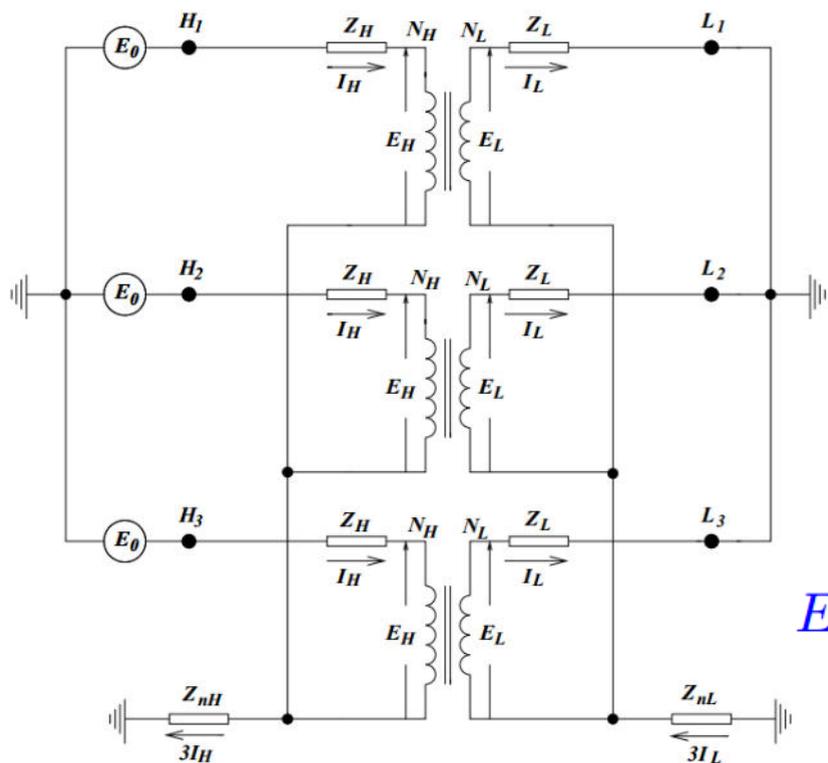
$$T = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$$

 $Z_+$ 
 $Z_-$ 
 $Z_o$ 

$$Z_S = \begin{bmatrix} Z_L & 0 & 0 \\ 0 & Z_L & 0 \\ 0 & 0 & Z_L + 3Z_n \end{bmatrix}$$

$$T^{-1} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}$$

# Transformador YY



$$E_L = E_H \left( \frac{N_L}{N_H} \right)$$

$$E_L = I_L (Z_L + 3Z_{nL})$$

$$I_L = I_H \left( \frac{N_H}{N_L} \right)$$

$$E_H \left( \frac{N_L}{N_H} \right) = I_H \left( \frac{N_H}{N_L} \right) (Z_L + 3Z_{nL})$$

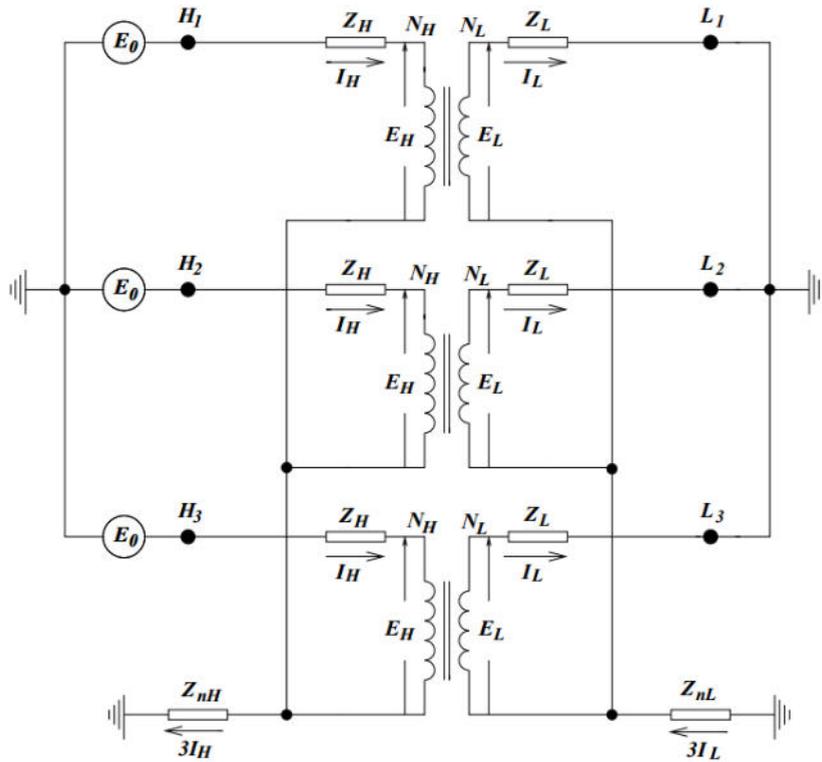
$$E_H = I_H \left( \frac{N_H}{N_L} \right)^2 (Z_L + 3Z_{nL})$$

$$E_0 = E_H + I_H (Z_H + 3Z_{nH})$$

$$E_0 = I_H (3Z_{nH} + Z_H + Z_L \left( \frac{N_H}{N_L} \right)^2 + 3Z_{nL} \left( \frac{N_H}{N_L} \right)^2)$$

$$z_0 = (3z_{nH} + z_{tr} + 3z_{nL})$$

# Transformador YY

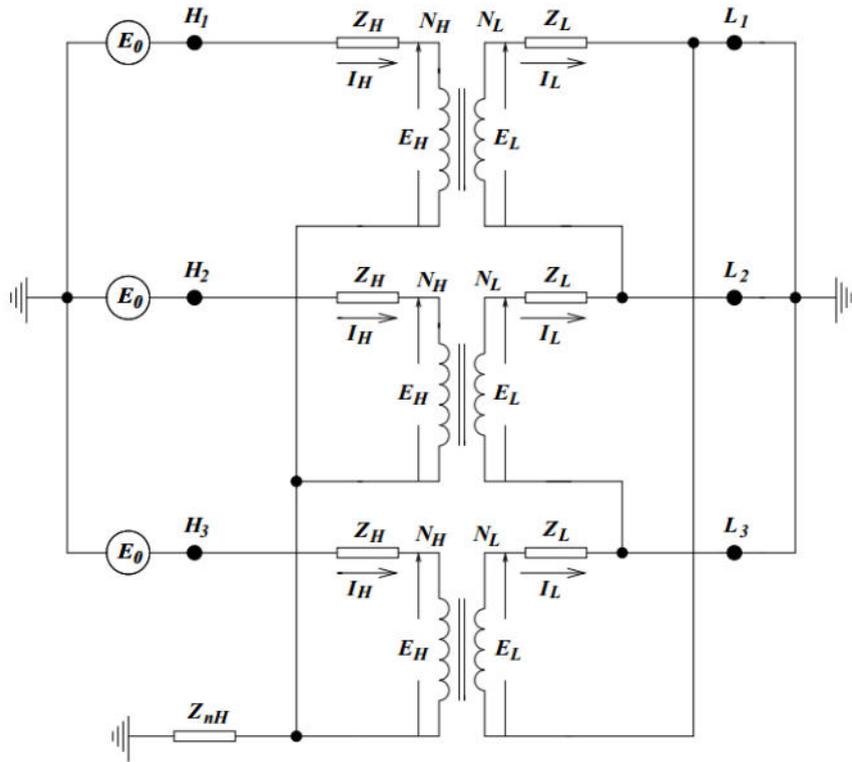


$$z_0 = (3z_{nH} + z_{tr} + 3z_{nL})$$

Sequência zero



# Transformador YΔ



$$I_L = I_H \left( \frac{N_H}{N_L} \right)$$

$$E_L = E_H \left( \frac{N_L}{N_H} \right)$$

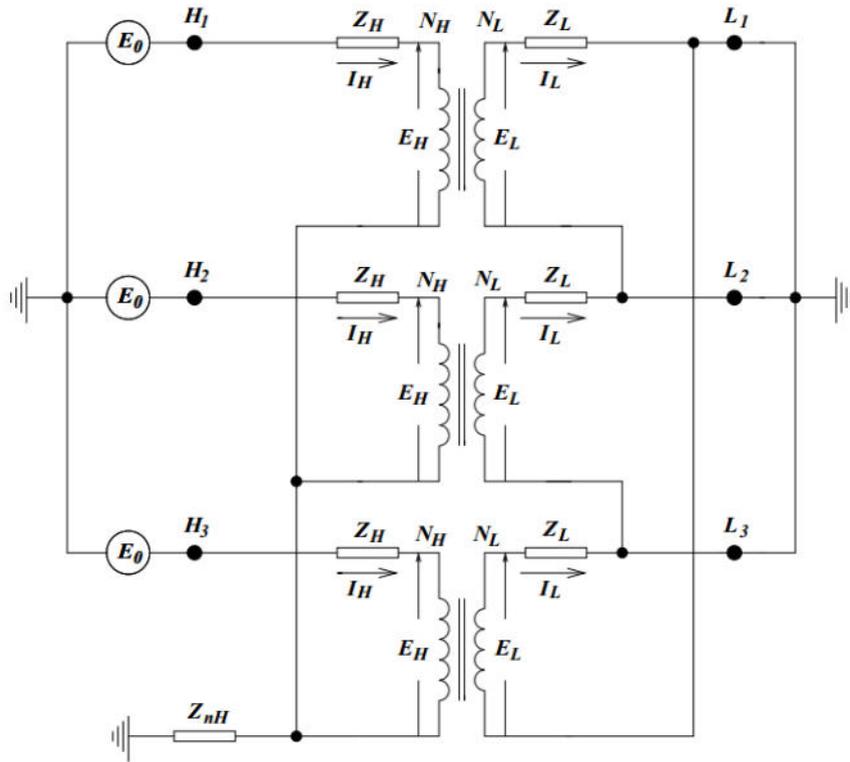
$$E_H \left( \frac{N_L}{N_H} \right) = I_H \left( \frac{N_H}{N_L} \right) Z_L$$

$$E_H = I_H \left( \frac{N_H}{N_L} \right)^2 Z_L$$

$$E_0 = E_H + I_H (Z_H + 3Z_{nH})$$

$$E_0 = I_H \left( 3Z_{nH} + Z_H + Z_L \left( \frac{N_H}{N_L} \right)^2 \right)$$

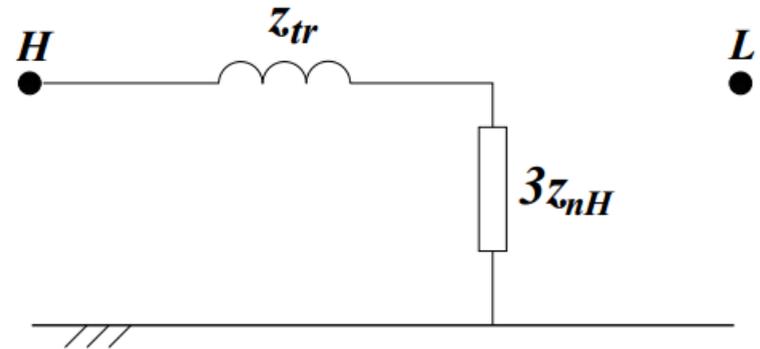
# Transformador YΔ



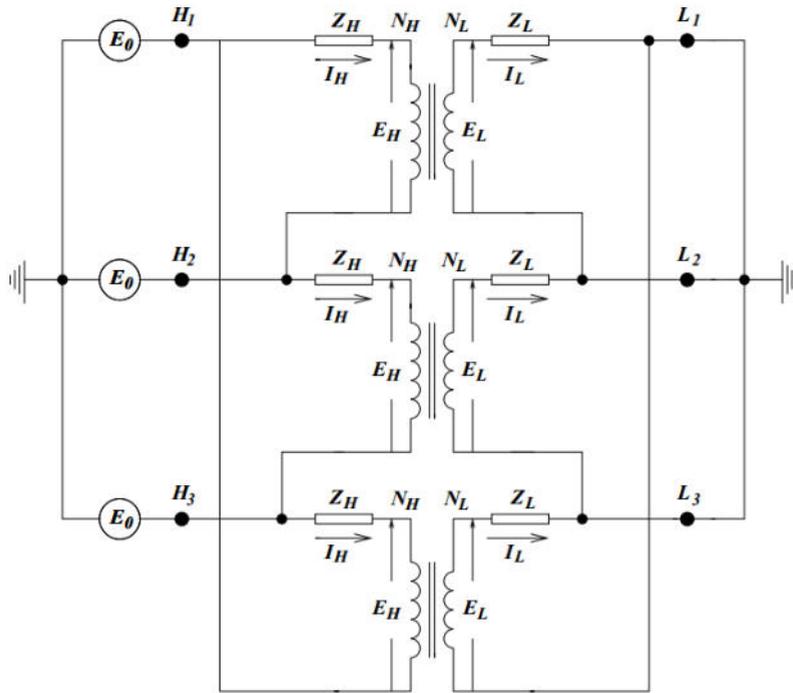
$$Z_0 = \frac{E_0}{I_H} = (3Z_{nH} + Z_H + Z_L \left(\frac{N_H}{N_L}\right)^2)$$

$$z_0 = (z_{tr} + 3z_{nH})$$

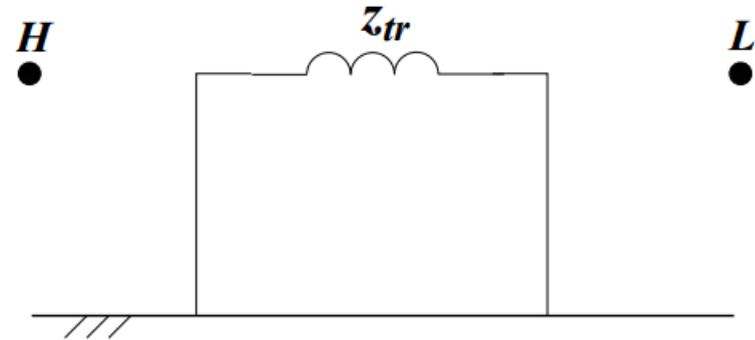
Sequência zero



# Transformador $\Delta\Delta$

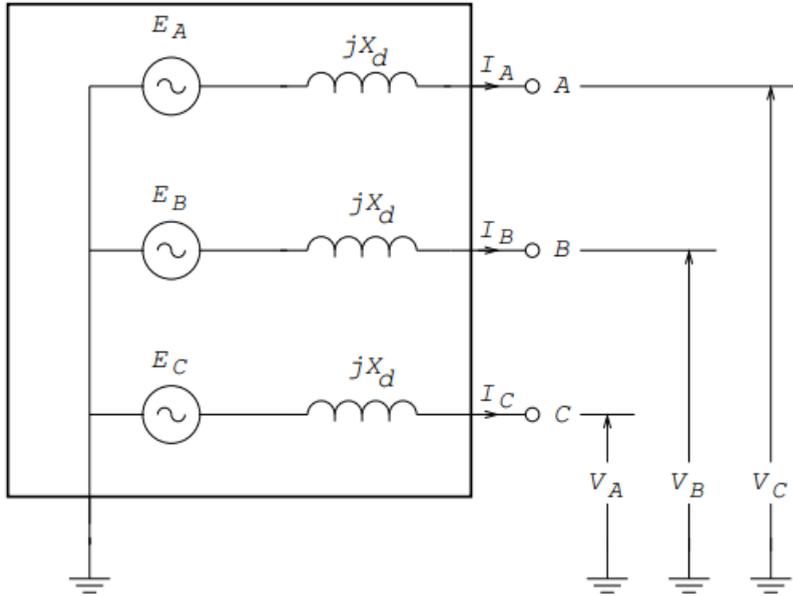


Sequência zero



# Máquina Síncrona

Condições equilibradas



$$\hat{V}_A = \hat{E}_A - j\hat{I}_A X_d$$

$$\hat{V}_B = \hat{E}_B - j\hat{I}_B X_d$$

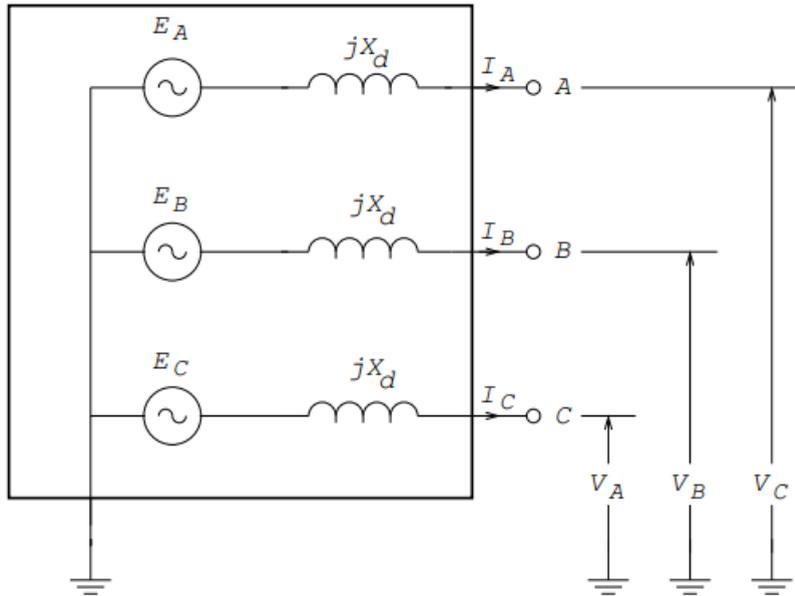
$$\hat{V}_C = \hat{E}_C - j\hat{I}_C X_d$$

$$\underline{V}_P = \underline{E}_P - Z_C \underline{I}_P$$

$$Z_C = \begin{bmatrix} jX_d & 0 & 0 \\ 0 & jX_d & 0 \\ 0 & 0 & jX_d \end{bmatrix}$$

# Máquina Síncrona

## Condições desequilibradas



$$\underline{Z}_C = \begin{bmatrix} Z_1 & Z_2 & Z_3 \\ Z_3 & Z_1 & Z_2 \\ Z_2 & Z_3 & Z_1 \end{bmatrix}$$

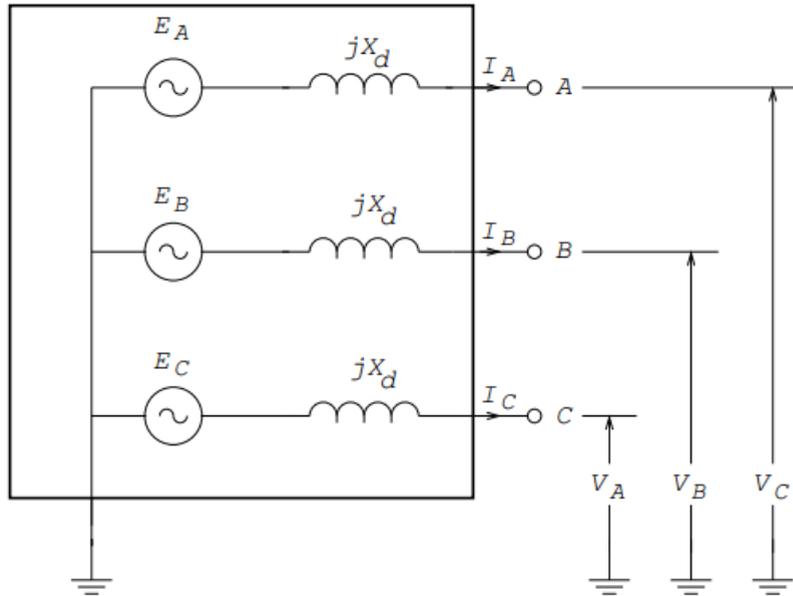
$$T\underline{V}_S = \underline{E}_P - \underline{Z}_C T\underline{I}_S$$

$$\underline{V}_S = T^{-1}\underline{E}_P - T^{-1}\underline{Z}_C T\underline{I}_S$$

$$\underline{V}_S = T^{-1}\underline{E}_P - \underline{Z}_S \underline{I}_S$$

# Máquina Síncrona

## Condições desequilibradas



$$\underline{V}_S = T^{-1} \underline{E}_P - Z_S \underline{I}_S$$

$$\underline{E}_P = \begin{bmatrix} \hat{E}_A \\ \hat{E}_B \\ \hat{E}_C \end{bmatrix} = \begin{bmatrix} \hat{E}_A \\ a^2 \hat{E}_A \\ a \hat{E}_A \end{bmatrix}$$

# Máquina Síncrona

## Condições desequilibradas

$$\underline{V}_S = T^{-1} \underline{E}_P - T^{-1} Z_C T \underline{I}_S$$

$$T^{-1} \underline{E}_P = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{E}_A \\ a^2 \hat{E}_A \\ a \hat{E}_A \end{bmatrix} = \frac{1}{3} \begin{bmatrix} \hat{E}_A(1 + a^3 + a^3) \\ \hat{E}_A(1 + a^4 + a^2) \\ \hat{E}_A(1 + a^2 + a) \end{bmatrix} = \begin{bmatrix} \hat{E}_A \\ 0 \\ 0 \end{bmatrix}$$

$$T^{-1} \begin{bmatrix} Z_1 & Z_2 & Z_3 \\ Z_3 & Z_1 & Z_2 \\ Z_2 & Z_3 & Z_1 \end{bmatrix} T = \begin{bmatrix} Z_1 + a^2 Z_2 + a Z_3 & 0 & 0 \\ 0 & Z_1 + a Z_2 + a^2 Z_3 & 0 \\ 0 & 0 & Z_1 + Z_2 + Z_3 \end{bmatrix}$$

 $Z_+$ 
 $Z_-$ 
 $Z_0$

# Máquina Síncrona

## Condições desequilibradas

$$\underline{V}_S = \boxed{T^{-1} \underline{E}_P} - Z_S \underline{I}_S$$

$$T^{-1} \underline{E}_P = \begin{bmatrix} \hat{E}_A \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{V}_{A+} = \hat{E}_A - \hat{I}_+ Z_+$$

$$\hat{V}_{A-} = 0 - \hat{I}_- Z_-$$

$$\hat{V}_{A0} = 0 - \hat{I}_0 Z_0$$



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Fotovoltaicos



<https://play.google.com/store/apps/details?id=br.developer.gesep.estimate>



# Obrigado!

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