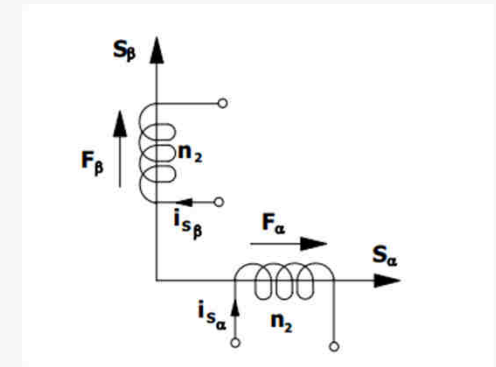
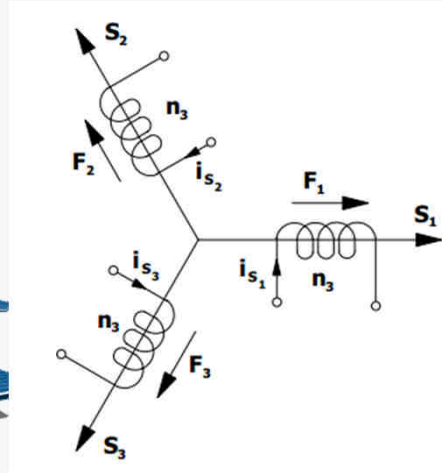
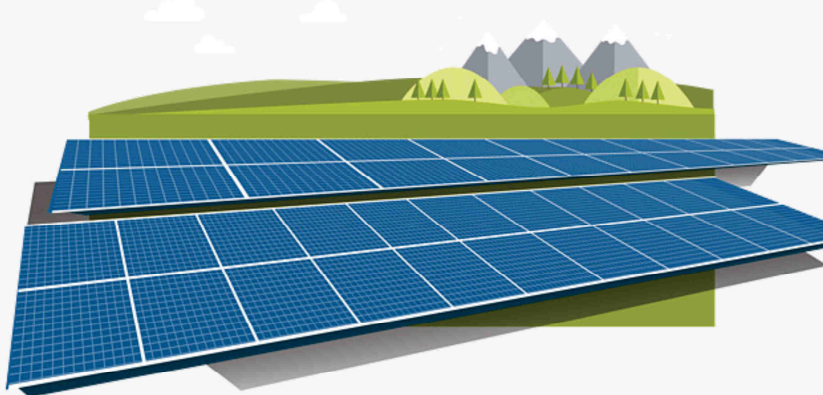


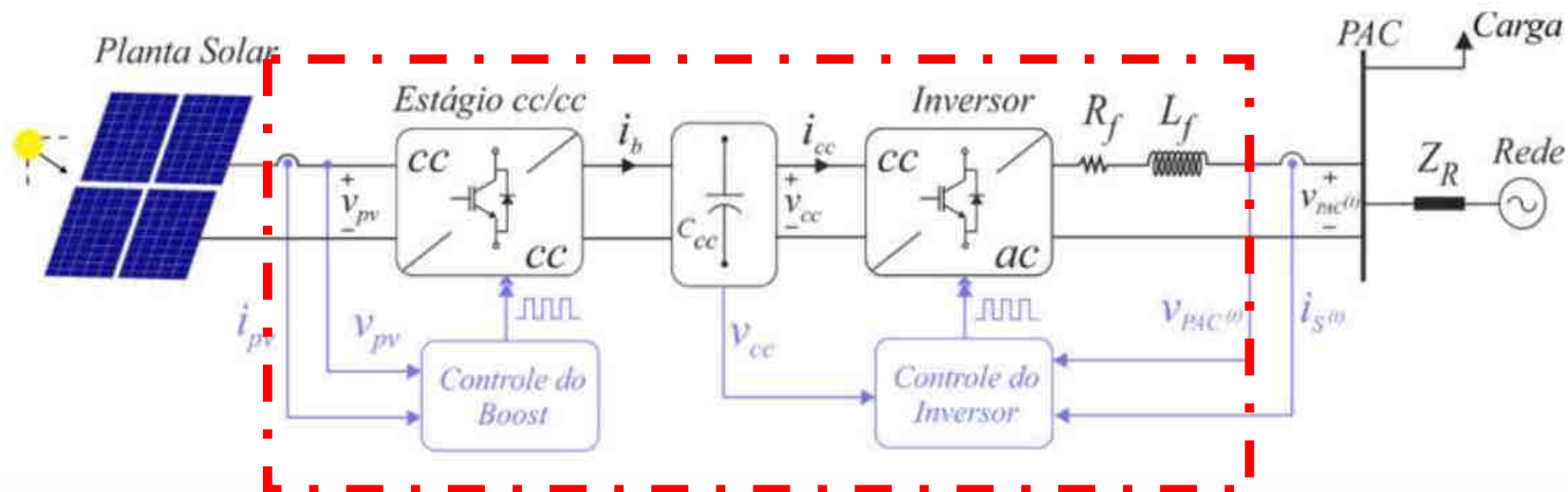
Modelagem e Controle de Sistemas Fotovoltaicos

Aula 06 – P1: Transformações de **Clarke** e de **Park**

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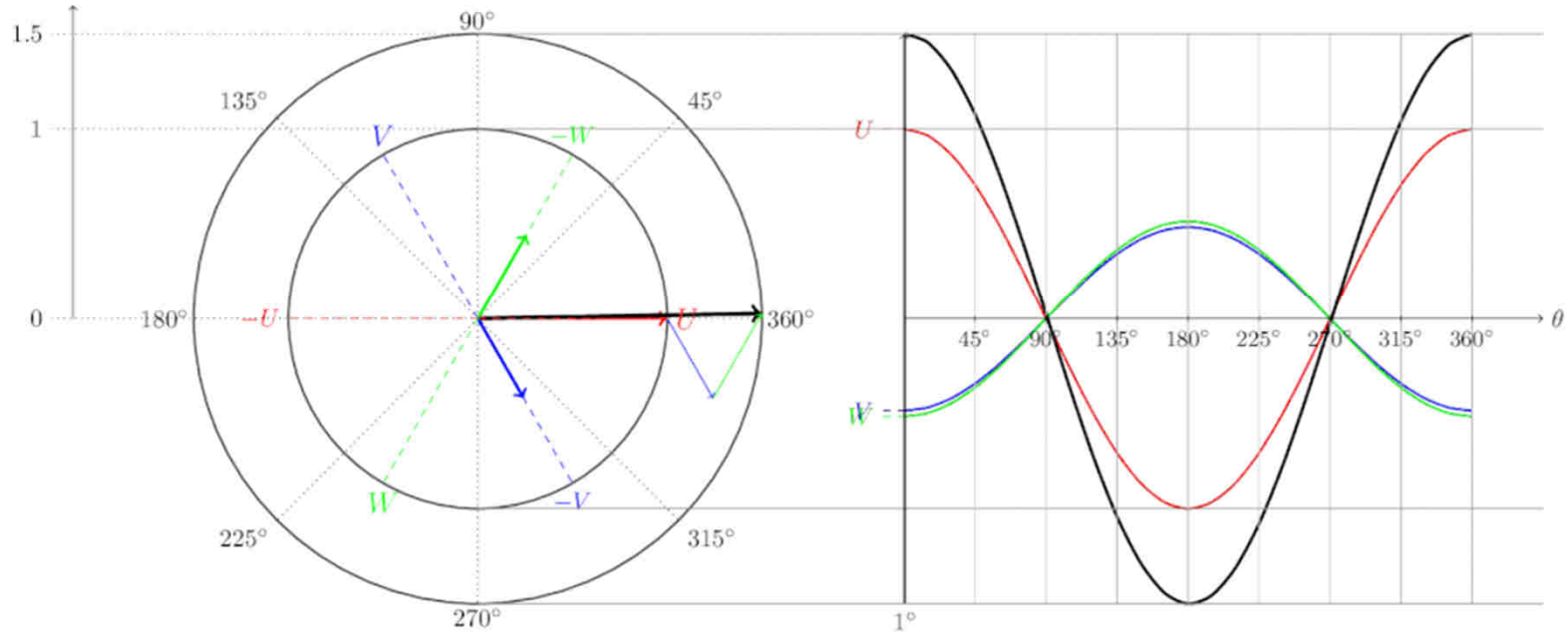


Inversor Fotovoltaico



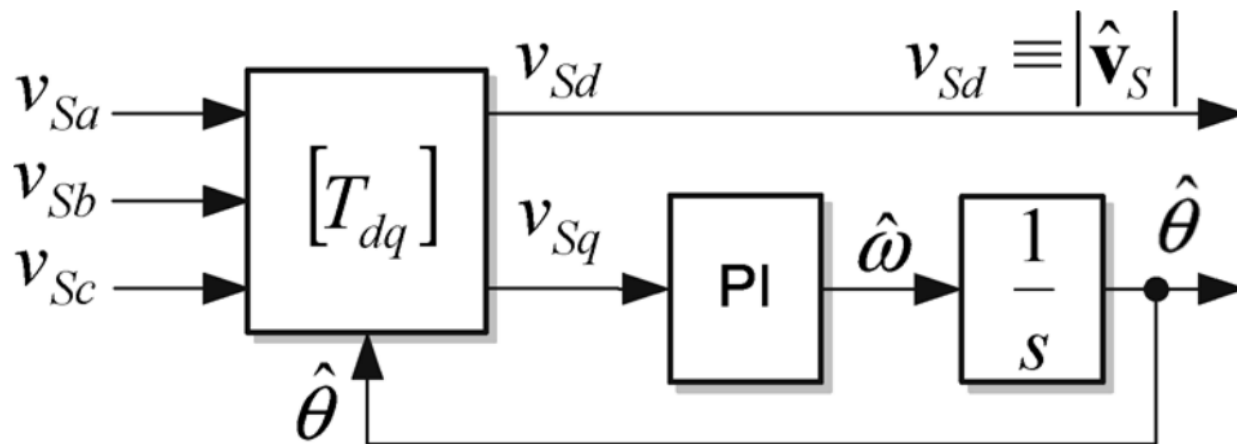
- ✓ Sincronismo com a rede
- ✓ Controlar a injeção de potência ativa e reativa na rede elétrica;
- ✓ Realiza a proteção do sistema fotovoltaico quando existem problemas na rede elétrica;

Three-phased sinusoidal system and its rotating equivalent space vector



Sistema de Sincronismo

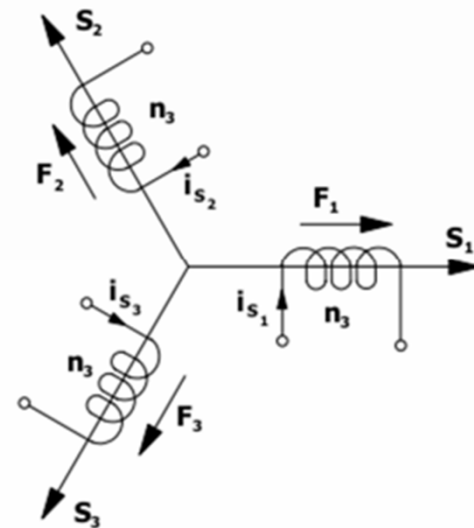
- ✓ PLL: *Phase Locked Loop*
- ✓ Medição do ângulo de fase e a frequência da rede
- ✓ Controlar a injeção de potência ativa e reativa na rede elétrica
- ✓ Realiza a proteção do sistema fotovoltaico quando existem problemas na rede elétrica



Fasor Espacial de um Sistema Trifásico

$$\begin{cases} i_a(t) = \hat{I} \cos(\omega_n t + \theta_0) \\ i_b(t) = \hat{I} \cos\left(\omega_n t + \theta_0 - \frac{2\pi}{3}\right) \\ i_c(t) = \hat{I} \cos\left(\omega_n t + \theta_0 - \frac{4\pi}{3}\right). \end{cases}$$

$$\begin{cases} |F_a(t)| = N_a |i_a(t)| \\ |F_b(t)| = N_b |i_b(t)| \\ |F_c(t)| = N_c |i_c(t)| \end{cases}$$



$$\vec{F}_R = \vec{F}_a + \vec{F}_b + \vec{F}_c.$$

$$\vec{F}_R = N_a i_a(t) e^{j0} + N_b i_b(t) e^{j\frac{2\pi}{3}} + N_c i_c(t) e^{j\frac{4\pi}{3}}$$

$$N = N_a = N_b = N_c$$

$$\vec{F}_R = N \left(i_a(t) e^{j0} + i_b(t) e^{j\frac{2\pi}{3}} + i_c(t) e^{j\frac{4\pi}{3}} \right) = N \vec{i}$$

Fasor Espacial de um Sistema Trifásico

$$\vec{F}_R = N \left(i_a(t) e^{j0} + i_b(t) e^{j\frac{2\pi}{3}} + i_c(t) e^{j\frac{4\pi}{3}} \right) = N \vec{i}$$

$$e^{j\theta} = \cos \theta + j \operatorname{sen} \theta$$

$$\vec{i} = \frac{3}{2} \hat{I} e^{j(\omega_n t + \theta_0)} \quad \longrightarrow \quad \text{Fasor Espacial}$$

$$\vec{i} = \frac{2}{3} \left(i_a(t) e^{j0} + i_b(t) e^{j\frac{2\pi}{3}} + i_c(t) e^{j\frac{4\pi}{3}} \right)$$

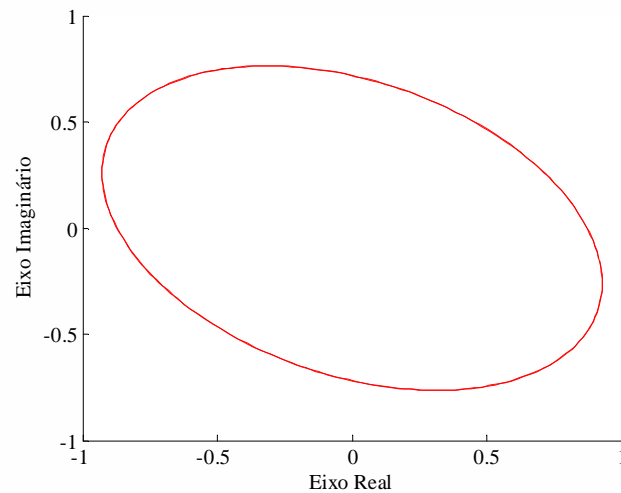
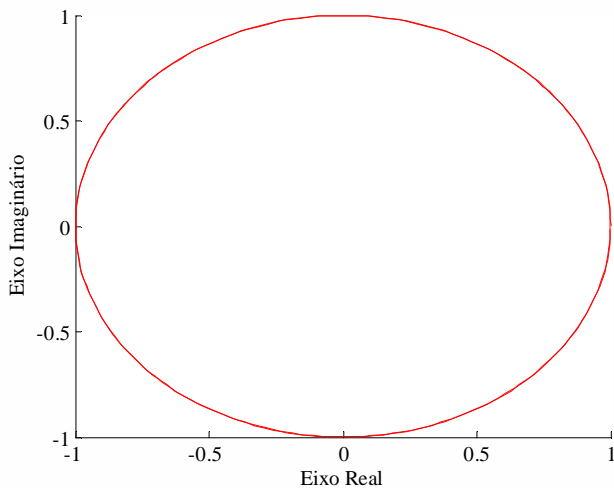
Invariância em amplitude da transformação.

$$\vec{v} = \frac{2}{3} \left(v_a(t) e^{j0} + v_b(t) e^{j\frac{2\pi}{3}} + v_c(t) e^{j\frac{4\pi}{3}} \right)$$

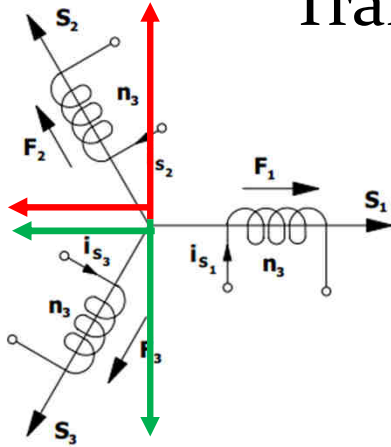
Fasor Espacial de um Sistema Trifásico

$$\begin{cases} i_a(t) = \hat{I}_a \cos(377 t) \\ i_b(t) = \hat{I}_b \cos\left(377 t - \frac{2\pi}{3}\right) \\ i_c(t) = \hat{I}_c \cos\left(377 t - \frac{4\pi}{3}\right). \end{cases}$$

a) $I_a = I_b = I_c = 1 \text{ p.u.}$
 b) $I_a = I_b = 1 \text{ p.u. e } I_c = 0,5 \text{ p.u.}$



Transformação de Clarke



$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix}.$$

$$F_\alpha = F_a + F_b \cos \frac{2\pi}{3} + F_c \cos \frac{4\pi}{3}$$

$$F_\beta = 0 + F_b \sin \frac{2\pi}{3} + F_c \sin \frac{4\pi}{3}$$

Transformação de Clarke

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = N_2 \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \text{ e } \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} = N_3 \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{N_3}{N_2} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

- ✓ Para que a transformação seja invertível, a matriz de transformação deve ser quadrada.
- ✓ Assim, define-se a corrente i_0 dada por:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = [T_{\alpha\beta}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$T_{0\alpha\beta} = \frac{N_3}{N_2} \begin{bmatrix} k & k & k \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

Transformação de Clarke

$$[T_{0\alpha\beta}][T_{0\alpha\beta}]^{-1} = [T_{0\alpha\beta}][T_{0\alpha\beta}]^T = I \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{pmatrix} \frac{N_3}{N_2} \end{pmatrix}^2 \begin{bmatrix} k & k & k \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} k & 1 & 0 \\ k & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ k & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

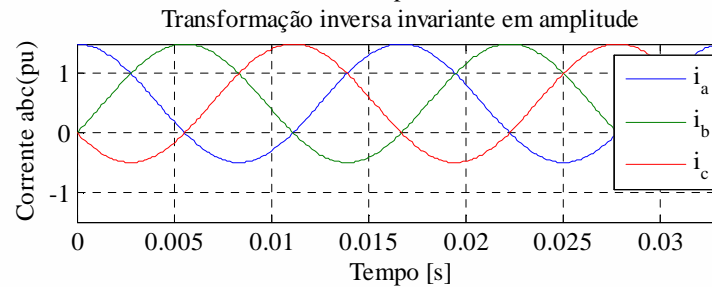
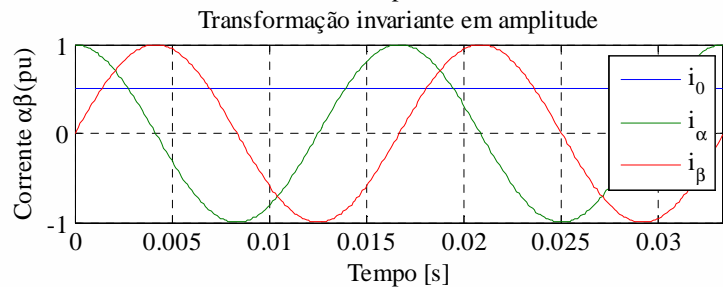
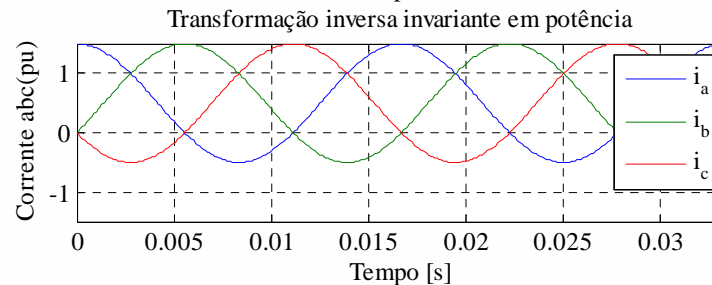
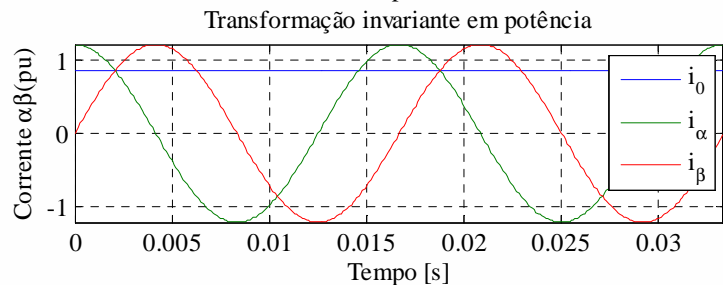
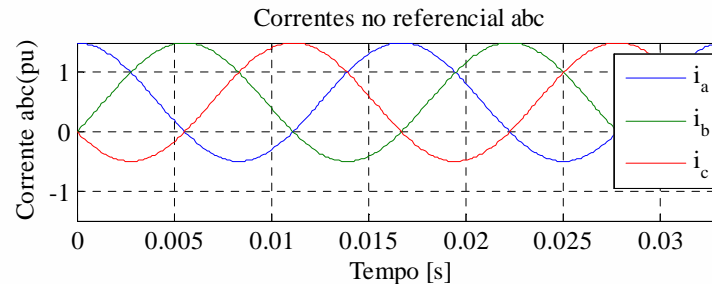
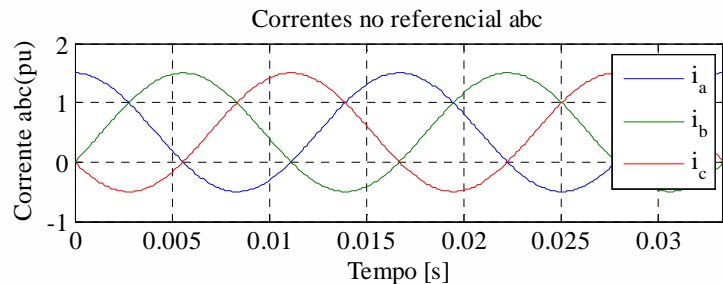
$$\begin{cases} 3 \left(\frac{N_3}{N_2} \right)^2 k^2 = 1 \\ \frac{3}{2} \left(\frac{N_3}{N_2} \right)^2 = 1 \end{cases} \quad \begin{cases} k = \frac{1}{\sqrt{2}} \\ \frac{N_3}{N_2} = \sqrt{\frac{2}{3}} \end{cases}.$$

Transformação de Clarke

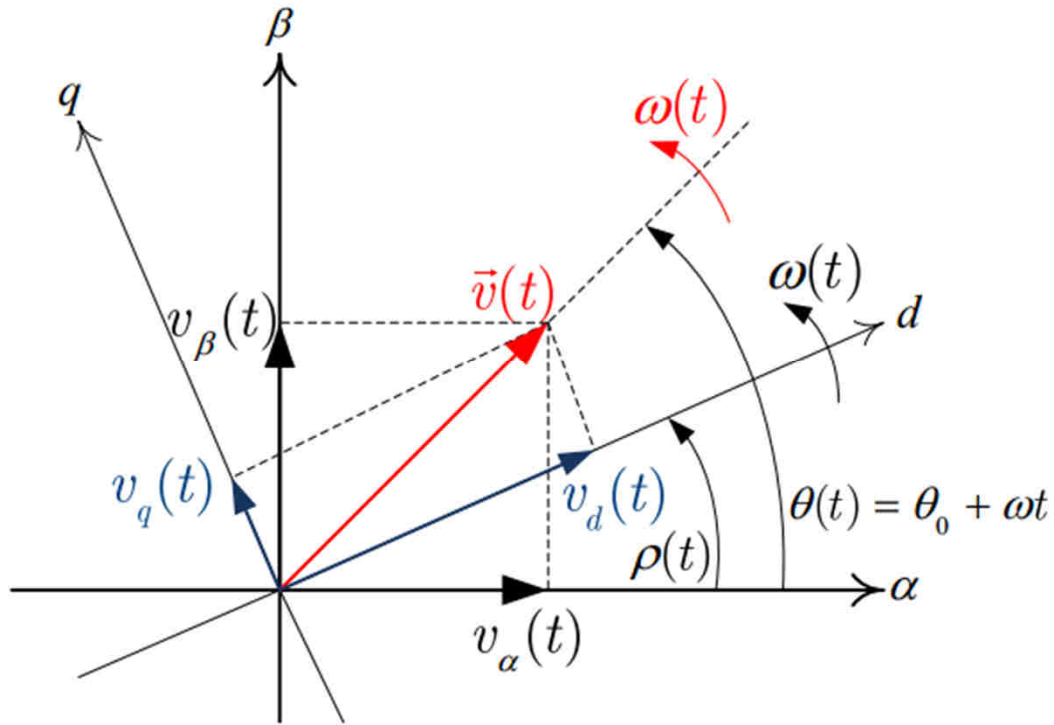
Esta transformação de coordenadas é conhecida como transformação de Clarke

$$[T_{0\alpha\beta}] = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad [T_{0\alpha\beta}]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Transformação de Clarke



Transformação de Park



$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos \rho & \sin \rho \\ -\sin \rho & \cos \rho \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

Transformação de Park

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \rho & \text{sen } \rho \\ 0 & -\text{sen } \rho & \cos \rho \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \quad \begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \rho & \text{sen } \rho \\ 0 & -\text{sen } \rho & \cos \rho \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}.$$

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \cos \rho & -\frac{1}{2} \cos \rho + \frac{\sqrt{3}}{2} \text{sen } \rho & -\frac{1}{2} \cos \rho - \frac{\sqrt{3}}{2} \text{sen } \rho \\ -\text{sen } \rho & \frac{1}{2} \cos \rho + \frac{\sqrt{3}}{2} \text{sen } \rho & \frac{1}{2} \cos \rho - \frac{\sqrt{3}}{2} \text{sen } \rho \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}.$$

Transformação de Park

$$\begin{cases} v_a(t) = \hat{V} \cos(\omega t + \theta_0) \\ v_b(t) = \hat{V} \cos\left(\omega t + \theta_0 - \frac{2\pi}{3}\right) \\ v_c(t) = \hat{V} \cos\left(\omega t + \theta_0 - \frac{4\pi}{3}\right) \end{cases}$$

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \cos \rho & \cos\left(\rho - \frac{2\pi}{3}\right) & \cos\left(\rho - \frac{4\pi}{3}\right) \\ -\text{sen } \rho & -\text{sen}\left(\rho - \frac{2\pi}{3}\right) & -\text{sen}\left(\rho - \frac{4\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \hat{V} \cos(\omega t + \theta_0) \\ \hat{V} \cos\left(\omega t + \theta_0 - \frac{2\pi}{3}\right) \\ \hat{V} \cos\left(\omega t + \theta_0 - \frac{4\pi}{3}\right) \end{bmatrix}$$

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{V} \cos(\omega t + \theta_0 - \rho) \\ \hat{V} \text{sen}(\omega t + \theta_0 - \rho) \end{bmatrix}$$

Transformação de Park

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ \widehat{V} \cos(\omega t + \theta_0 - \rho) \\ \widehat{V} \sin(\omega t + \theta_0 - \rho) \end{bmatrix}$$

$$\vec{v}_{dq} = \vec{v} e^{-j\rho(t)} = \widehat{V} e^{-j(\{\omega t - \rho(t)\} + \theta_0)}$$

$$\rho(t) = \omega t$$

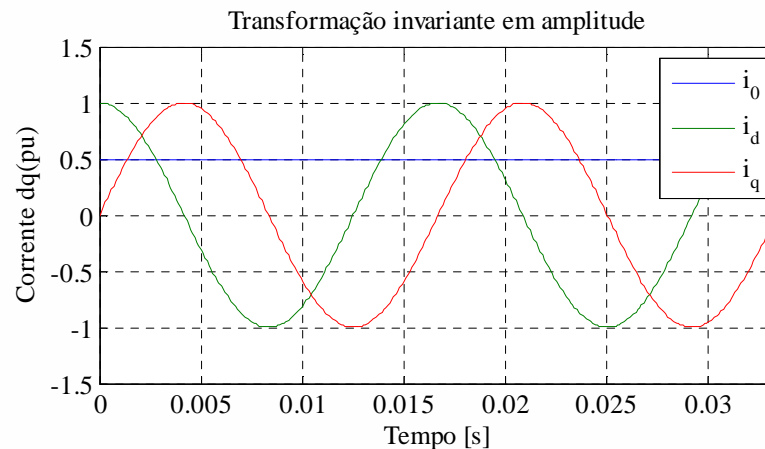
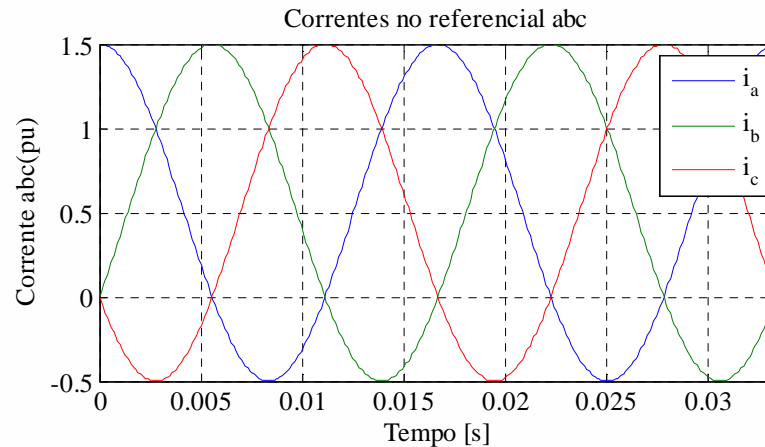
$$\vec{v}_{dq} = \widehat{V} e^{-j(\{\omega t - \omega t\} + \theta_0)} \Rightarrow \boxed{\vec{v}_{dq} = \widehat{V} e^{-j\theta_0}}$$

$$\vec{v}_{dq} = v_d + j v_q \cdot \begin{cases} v_d = \widehat{V} \cos(\theta_0) \\ v_q = \widehat{V} \sin(\theta_0) \end{cases}$$

Transformação de Park

$$\begin{cases} i_a(t) = \cos(377 t + \theta) \\ i_b(t) = \cos\left(377 t - \frac{2\pi}{3} + \theta\right) \\ i_c(t) = \cos\left(377 t - \frac{4\pi}{3} + \theta\right) \end{cases}$$

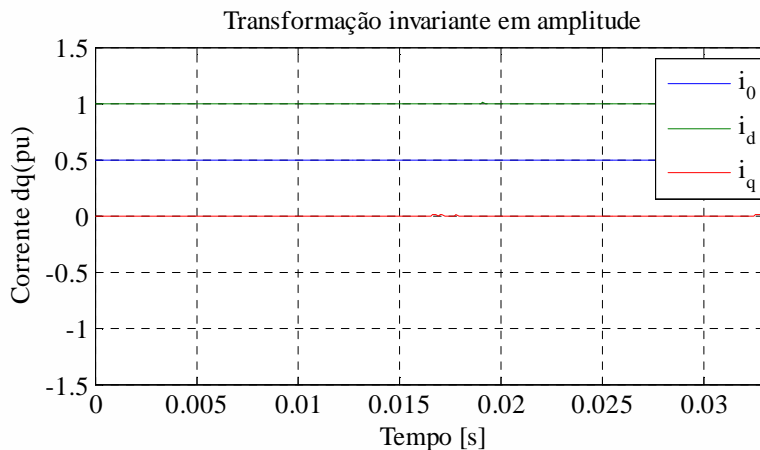
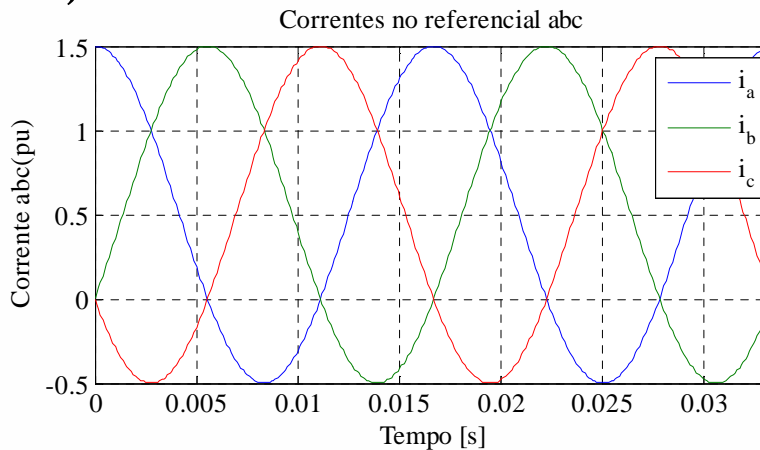
a) $\rho = 0$ e $\theta = 0^\circ$. Qual a amplitude das componentes obtidas? Qual a frequência das formas de onda obtidas?



Transformação de Park

$$\begin{cases} i_a(t) = \cos(377t + \theta) \\ i_b(t) = \cos\left(377t - \frac{2\pi}{3} + \theta\right) \\ i_c(t) = \cos\left(377t - \frac{4\pi}{3} + \theta\right) \end{cases}$$

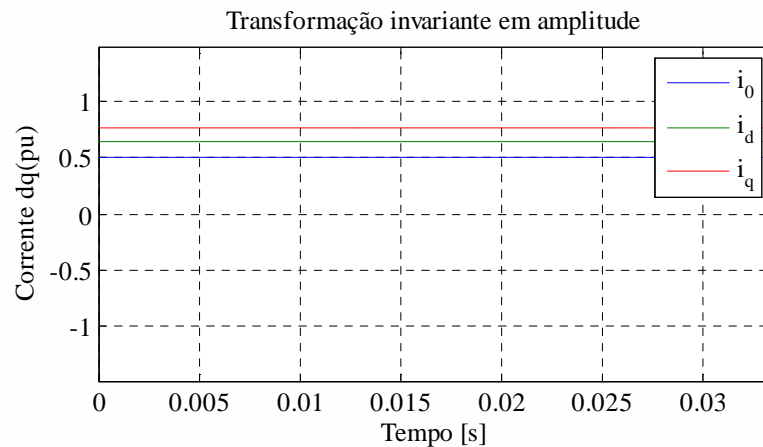
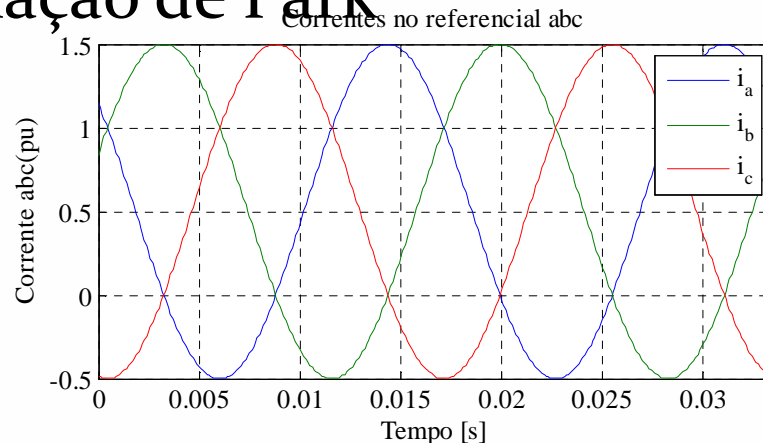
b) $\rho = 377t$ e $\theta = 0^\circ$. Qual a amplitude das componentes obtidas? Qual a frequência das formas de onda obtidas?



Transformação de Park

$$\begin{cases} i_a(t) = \cos(377t + \theta) \\ i_b(t) = \cos\left(377t - \frac{2\pi}{3} + \theta\right) \\ i_c(t) = \cos\left(377t - \frac{4\pi}{3} + \theta\right) \end{cases}$$

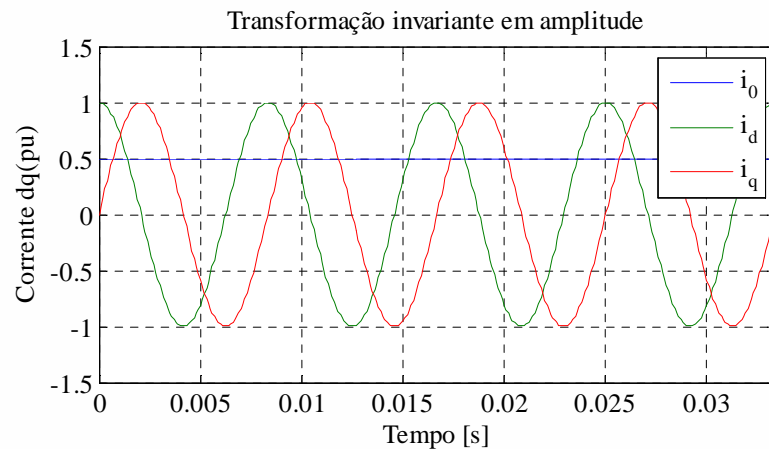
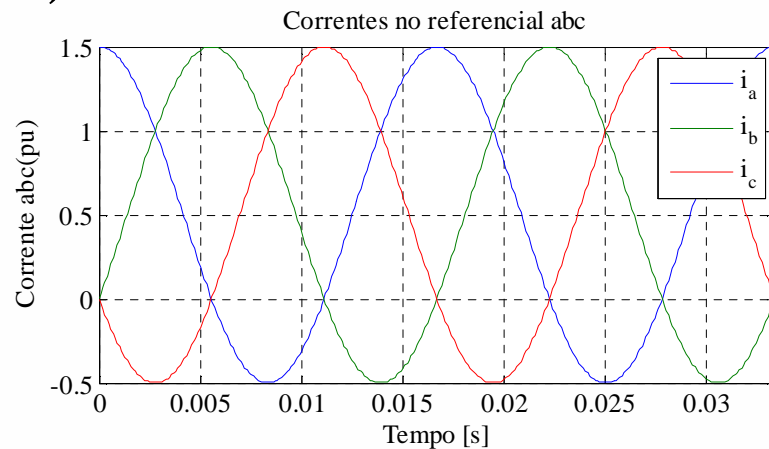
c) $\rho = 377t$ e $\theta = 50^\circ$. Qual a amplitude das componentes obtidas? Qual a frequência das formas de onda obtidas? Conclua.



Transformação de Park

$$\begin{cases} i_a(t) = \cos(377t + \theta) \\ i_b(t) = \cos\left(377t - \frac{2\pi}{3} + \theta\right) \\ i_c(t) = \cos\left(377t - \frac{4\pi}{3} + \theta\right) \end{cases}$$

d) $\rho = -377t$ e $\theta = 0^\circ$. Qual a amplitude das componentes obtidas? Qual a frequência das formas de onda obtidas? Conclua.





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<https://play.google.com/store/apps/details?id=br.developer.gesep.estimate>



Obrigado!

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